we obtain the coefficients $A_{2j,m}$ $(j = 0, 1, 2, \ldots, N;\ m = 0, 1, 2, \ldots, M)$ of the expansion (5.12) of the function $A_{2j}(t)$ $(j = 0, 1, 2, \ldots, N)$ in Laguerre polynomials. Here N is determined by the order of the truncated system (4.8), and M is the number of terms retained in series (5.12).

Graphs of the function $A_0(t)$ are represented in Fig.2 for the values $g = 1, 5, 10$ (the continuous, dashed, and dash-dot lines, respectively) for $T = \pi/2$, $\beta = 0.535$, $\zeta_1 = 0.927$ and diagrams of the contact stresses are displayed for different values of t at $g = 5$. Curve *1* corresponds to the value $t = 0.1$, curve *2* to the value $t = 2.6$; the stress diagrams for $t = 0.6$ and 3.6 (curve *3*), for $t = 1.1$ and 3.1 (curve *4*), and also for $t = 1.6$ and 2.1 (curve *5*) agree practically in pairs.

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THERMOPHORESIS AND THE INTERACTION OF UNIFORMLY HEATED SPHERICAL PARTICLES IN A GAS*

A.YU. BORIS

The thermophoresis of a uniformly heated spherical particle caused by the action of Branett temperature stresses is investigated, and the thermophoretic force is calculated for arbitrary temperature drops between the particle and the gas. An analogous problem was considered earlier /1/ in the linear approximation of a small temperature drop.

The results obtained are used to estimate the nature and interaction force of widely spaced particles. It is shown that the gas motion caused by the temperature stresses can result in displacement of the system of differently heated particles.

We consider a uniformly heated (cooled) spherical particle in a gas at rest at infinity whose temperature varies weakly along the x axis. The gas is regarded as a continuous medium. The temperature stresses evoke a pressure redistribution and gas motion around the particles /2/, which will result in the appearance of a thermophoretic force acting on the particle.

We introduce dimensionless coordinates, temperature, density, viscosity, thermal conductivity, velocity, pressure, and force as follows:

$$a\,(x\ y\ z),\quad T_\infty T,\quad \rho_\infty\rho\quad \mu_\infty\mu,\quad \lambda_\infty\lambda$$

$$\frac{\mu_\infty}{\rho_\infty a}\mathbf{v},\quad \frac{P}{P_\infty} = 1 + \left[\frac{\mu_\infty}{\rho_\infty a}(RT_\infty)^{-1/2}\right]p,\quad \frac{\mu_\infty^2}{\rho_\infty}\mathbf{F}$$

Here a is the radius of the sphere; when there is no temperature gradient at infinity, the subscript ∞ is ascribed to the appropriate gas parameters far from the sphere. The dimensionless continuity, energy, and momentum equations describing the flow around the particle /2/, and the boundary conditions can be written in the following form:

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$$\nabla \mathbf{v} = \mathbf{v}\nabla \ln T \tag{1}$$
$$E\mathbf{v}\nabla \ln T = \nabla (T^s\nabla T),\ E = {}^5/_2 (\varkappa - 1)\,\mathrm{Pr}/\varkappa,\ \varkappa = c_p/c_v \tag{2}$$
$$T^{-1} (\mathbf{v}\nabla)\,\mathbf{v} + \nabla\Pi = \Pi^{(1)} + \alpha_1 T^{2s-2} (\nabla T)^2\nabla T + \alpha_2(\mathbf{v}\nabla \ln T)\,\nabla (T^s) \tag{3}$$
$$\Pi = p + {}^2/_3 (1 + E\omega_1)\, T^{s-1} (\mathbf{v}\nabla T) + \alpha_3 T^{2s-1}(\nabla T)^2$$
$$\Pi_i^{(1)} = \frac{\partial}{\partial x_j}\left[T^s\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\right]$$
$$\alpha_1 = -{}^1/_2 (s\omega_1 + \omega_3),\ \alpha_2 = E\,(\omega_1 + \omega_3/s),\ \alpha_3 = {}^1/_2 (s\omega_1 - {}^1/_3\omega_3)$$
$$\mathbf{v} = 0,\ \ T = T_w = \mathrm{const}\ \ (r = 1) \tag{4}$$
$$\mathbf{v} \to 0,\ \ T \to 1 + \varepsilon\cos\theta\ \ (r \to \infty),\ \ \varepsilon = (\nabla T)_\infty \ll 1 \tag{5}$$

Here Pr is the Prandtl number, $\varkappa$ is the ratio of the specific heats (for a monatomic gas $\mathrm{Pr} = {}^2/_3$, and $\varkappa = {}^5/_3$). The density is deliminated by using the equation of state $\rho = T^{-1}$. A power-law dependence of the transport coefficient on the temperature $\mu = \lambda = T^s$ is taken. Terms expressing viscous energy dissipation (negligibly small in the case of slow flows) are neglected in the energy equation (2). The Barnett temperature stresses are taken into account in the momentum equation (3); ω_1 and ω_3 are coefficients for the Barnett terms in the stress tensor ($\omega_1 = 3,\ \omega_3 = 0$ for $s = 1$, and $\omega_1 = 2.418,\ \omega_3 = 0.99$ for $s = {}^1/_2$).

Adhesion conditions for the velocity and temperature (4) are taken on the sphere surface. There is not the usual slip of the gas along the surface since $T_w = \mathrm{const}$. Second-order slip in the Knudsen number /3/ and temperature jumps on the surface are not taken into account since they yield small corrections of the order of the Knudsen number, to the main solution. Far from the sphere the gas is at rest, and there is a small temperature gradient (5).

We note that when using the Navier-Stokes equations (there are no temperature stresses in (3)), the thermophoretic force is zero for these boundary conditions since classical thermophoresis occurs because of slip of the gas along the inhomogeneously heated particle surface ($T_w \neq \mathrm{const}$).

The thermophoretic force occurring because of the pressure redistribution caused by the temperature stresses was calculated in /1/ in the linear approximation in ΔT in the case of a weakly heated particle ($\Delta T = T_w - 1 \ll 1$) (the gas motion was not taken into account since $v = O(\varepsilon\Delta T^2),\ p = O(\varepsilon\Delta T)$).

Let us examine the case of arbitrary temperature drops ($\Delta T \sim 1$) when the gas motion must be taken into account. Near the sphere the small temperature gradient at infinity evokes only a perturbation of the spherically-symmetric temperature and pressure distributions. Consequently, we seek a solution in the form of an expansion in ε:

$$T = T_0 (r) + \varepsilon T_1 (r, \theta) + \ldots,\ \ \mathbf{v} = \mathbf{v}_0 (r) + \varepsilon\mathbf{v}_1 (r, \theta) + \ldots,\ \ \Pi = \Pi_0 (r) + \varepsilon\Pi_1 (r, \theta) + \ldots \tag{6}$$
$$T_0 = [1 + (T_w^{s+1} - 1)\, r^{-1}]^{1/(s+1)},\ \ \mathbf{v}_0 = 0,\ \ \Pi_0 = \Pi_0 (r) \tag{7}$$

Here (7) is the solution of (1)–(3) for a uniformly heated sphere when there is no temperature gradient at infinity ($\varepsilon = 0$). The temperature stresses are equilibrated by the pressure /2/ (the specific form of the dependence $\Pi_0(r)$ is not required later), and there is no force.

For $r \sim \varepsilon^{-1}$ expansion (6) could become incorrect since the convective terms in the energy equation, discarded in obtaining T_0 in (7), become of the same order as the rest. From (5) and (7) we have an estimate of the decrease in T for large r: $T = 1 + O(\varepsilon r) + O(r^{-1})$.

To estimate the order of decrease in velocity at large distances from the particle we use the solution for a point force /4/. Since the force acting on the sphere is obviously proportional to ε, we have the following estimate for velocities at large r: $v = O(\varepsilon r^{-1})$. Therefore, we obtain for $r \sim \varepsilon^{-1}$

$$\nabla (T^s\Delta T) = O(r^{-3}) + O(\varepsilon r^{-2}) = O(\varepsilon^3)$$
$$T^{-1} (\mathbf{v}\nabla T) = O(\varepsilon r^{-3}) + O(\varepsilon^2 r^{-1}) = O(\varepsilon^3)$$

Since all the terms in the energy equation will be of the order of ε^3 for $r \sim \varepsilon^{-1}$, and expansion (6) is considered to terms of order ε, of expansion (6) holds in the whole flow domain. Therefore, the boundary conditions for the perturbed quantities have the form

$$\mathbf{v}_1 = 0,\ T_1 = 0\ \ (r = 1),\ \ \mathbf{v}_1 \to 0,\ T_1 \to r\cos\theta\ (r \to \infty) \tag{8}$$

The equations for $\mathbf{v}_1, T_1, \Pi_1$ are obtained in an obvious way by substituting expansion (6) into (1)–(3) and linearizing with respect to ε.

The form of the boundary conditions (8) enables the variables ($v_{1r} = f(r)\cos\theta,\ v_{1\theta} = -g(r)\sin\theta$, $\Pi_1 = h(r)\cos\theta + n(r),\ T_1 = T_0^{-s}[\tau(r)\cos\theta + m(r)]$) to be separated and enables the system for the perturbed quantities to be reduced to a system of ordinary differential equations in f, g, h, τ with coefficients dependent on T_0 (an analgous system is presented in /5/). The boundary conditions take the following form:

$$f = 0,\ \ g = 0,\ \ \tau = 0\ \ (r = 1),\ \ f \to 0,\ \ g \to 0,\ \ \tau \to r\ \ (r \to \infty)$$

The appropriate boundary value problem was solved numerically by the method of orthogonal factorization /6/. The expression for the force acting on the particle can be obtained by integrating the stresses (including the temperature stresses too) over the surface of the sphere

$$F = {}^4/_3\pi\, [2T_w{}^s g'(1) - h(1) + (s\omega_1 + \omega_3)\, T_w^{s-1} T_0'(1)\, \tau'(1)]\, \varepsilon$$

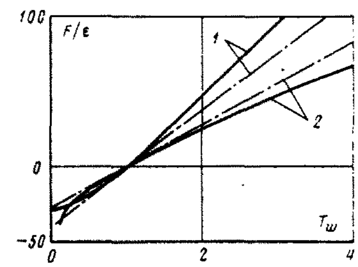
Let us examine the results of computations for the thermophoretic force shown in the figure for two laws of the temperature dependence of the transfer coefficients: $s = 1$ (curve *1*) and $s = {}^1/_2$ (curve *2*), where the results obtained in /1/ are shown by dashes ($F = 4\pi\,(s\omega_1 + \omega_3)\, \Delta T \varepsilon$). It is seen that a linear approximation in ΔT yields an almost exact value of the force even for $\Delta T \sim 1$. Taking account of the non-linearity is felt only for sufficiently high temperature drops, where the force for $s = 1$ grows in comparison to the linear approximation and diminishes for $s = {}^1/_2$.

We will now use the results obtained to estimate the force and nature of the interaction between two uniformly heated particles separated by a large distance R. The particle radii are a_i and the temperatures are T_{wi}. A system of Cartesian coordinates x_i, y_i, z_i is associated with the centre of each particle (the x_i axes are along the line connecting the centres of the particles), and the radius-vector r_i. We shall assume that $R \gg a_i$, we take the radius of the first particle as the characteristic dimension (the particle radii are considered to be quantities of the same order of magnitude), and we introduce the small parameter $\varepsilon_R = a_1/R$.

Let us examine the temperature field produced by the second particle in the neighbourhood of the first. Using solution (7), we obtain for the temperature field of the second particle in a coordinate system connected to the first particle after expanding in series in ε_R:

$$T_{02} = [1 + (T_{w2}^{s+1} - 1)\, r_2{}^{-1}]^{1/(s+1)}, \quad r_2 = [(R - x_1)^2 + y_1{}^2 + z_1{}^2]^{1/2} \tag{9}$$
$$T_{02} = 1 + \varepsilon_R D_2 + \varepsilon_R{}^2 D_2 x_1 + O(\varepsilon_R{}^3), \; D_2 = a_2 a_1{}^{-1} (T_{w2}^{s+1} - 1)/(s+1)$$

The second term in (9) yields a small addition to the uniform temperature field, which does not influence the force acting on the particle in the fundamental approximation, and the gas motion around the particle, caused by the inhomogeneity of the temperature distribution, and, consequently, it need not be taken into account later. The third term in (9), proportional to $\varepsilon_R{}^2$, yields a homogeneous temperature gradient directed along the line connecting the centres of the particles. The inhomogeneity of the temperature will result in gas motion around the first particle at velocities of the order of $\varepsilon_R{}^2$, due to the action of the temperature stresses. The first particle causes an analogous inhomogeneity of order $\varepsilon_R{}^2$ in the temperature field near the second particle, hence the temperature stresses will cause a flow with velocities of the same order near the second particle. We use the solution for a point force $v = O(r_2{}^{-1})$ /4/ to estimate the decrease in the velocity. Therefore, the velocity perturbation caused by the second particle in the neighbourhood of the first is of the order of $\varepsilon_R{}^3$ and can be neglected.



Therefore, to terms $O(\varepsilon_R{}^3)$ the problem of the action of a second particle on the first reduces to the problem examined above on the thermophoresis of the first particle in the temperature field of the second particle with a gradient determined by (9). Hence, by using the resutls obtained earlier for the magnitudes of the forces acting on the first and second particles, respectively, we have

$$F_{12} = F(s, T_{w1})\, \varepsilon_R{}^2 D_2 = F(s, T_{w1})\, \frac{T_{w2}^{s+1} - 1}{s+1}\, \frac{a_1 a_2}{R^2}$$
$$F_{21} = F(s, T_{w2})\, \frac{T_{w1}^{s+1} - 1}{s+1}\, \frac{a_1 a_2}{R^2}$$

The function $F(s, T_w)$ is shown in the figure by solid lines for $s = 1$ and $s = {}^1/_2$. In the case of small temperature drops $(\Delta T_i = T_{wi} - 1 \ll 1)$, the expressions for the forces are analogous to those obtained in /1/ from the electrostatic analogy $(F_{12} = F_{21} = 4\pi\,(s\omega_1 + \omega_3)\, \Delta T_1 \Delta T_2 a_1 a_2 R^{-2})$. For arbitrary pressure drops $(\Delta T_i \sim 1,\ \Delta T_1 \neq \Delta T_2)$ the gas flows caused by the temperature stresses around the particles can result in $F_{12} \neq F_{21}$. Therefore, particles will not only be attracted to each other $(\Delta T_1 \cdot \Delta T_2 < 0)$ or repulsed $(\Delta T_1 \cdot \Delta T_2 > 0)$ /1/ but can still be shifted as a whole. The direction of the shift will be determined by the relationship of F_{12} to F_{21}. Using the graphs presented in the figure for F, it can be shown that the sign of the displacement (a shift from the first particle towards the second is selected as positive) agrees with the sign of the expression $(T_{w1} - 1)(T_{w2} - 1)(T_{w2} - T_{w1})$. The force causing this shift $\Delta F = F_{12} - F_{21}$ is due to the gas motion between particles, caused by the action of the temperature stresses.

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ON SPATIALLY HOMOGENEOUS RELAXATION IN THE DOMAIN OF HIGH MOLECULAR VELOCITIES*

A.A. ABRAMOV

Asymptotic solutions of the Boltzmann equation are studied for the spatially homogeneous relaxation of the distribution function in the domain of fast molecules, as well as the evolution of the perturbations in the distribution function in the case of sherical scattering motion /1/. The problem was studied in the linear approximation in /2/, where exact solutions of the linearized Boltzmann equation were obtained for a specified form of molecular collision cross-sections.

Let us consider a spatially homogeneous gas composed of molecules, regarded as rigid spheres. We shall assume that at the initial instant we specify, on the range (ξ_{1*}, ξ_{2*}) of velocities,

$$\xi_{1*} \gg c = (2kT/m)^{1/2},\ \Delta\xi = \xi_{2*} - \xi_{1*} \leqslant O\,(c^2/\xi_{1*})$$

a spatially homogeneous perturbation of the distribution function $\Phi(\xi)$ relative to the Maxwell distribution $f_0 = n(\pi c^2)^{-3/2}\exp(-\xi^2/c^2)$, i.e. $f = f_0 + \Phi$. We shall require that the following relation holds:

$$f_m = \max_{(\xi_{1*},\, \xi_{2*})} \Phi(\xi) = O\,[f_0(\xi_{1*})] \tag{1}$$

and we shall have to explain how this perturbation evolves with time.

It was shown in /3, 4/ that the integral of elastic collisions $J(f, f)$ exhibits the following asymptotic behaviour at large velocities $\xi \gg c$:

$$J(f, f) = \int (f_1'f' - ff_{01})\, g\, d\sigma\, d\xi_1, \quad g = |\xi_1 - \xi_2| \tag{2}$$

since the fast molecules $(\xi \gg c)$ collide mainly with the "thermal" molecules moving with velocity of order c.

We find that the collision integral (2) can be simplified for the problem in question.

We shall denote the thermal molecules by X and the fast molecules by Γ. When the fast and thermal molecules (Γ, X) collide, we can have the molecules in the following states (Γ, X), (Γ, Γ), (X, Γ). Analyzing the dynamics of molecular collisions, with the molecules treated as rigid spheres, we can show that when $\xi \in (\xi_{1*}, \xi_{2*})$ we can neglect, within the range of collisions (2), the "glancing" $(\Gamma, X) \to (\Gamma, X)$ and "frontal" $(\Gamma, X) \to (X, \Gamma)$ collisions with an error of order $O\,(c^2/\xi^2)$, as $\xi \to \infty$.

Consider the product

$$f_1'f' = f_{01}f_0\left[1 + \frac{\delta f'}{f_0'} + \frac{\delta f_1'}{f_{01}'} + \frac{\delta f'\delta f_1'}{f_0'f_{01}'}\right]$$
$$(\delta f' = f' - f_0',\ \delta f_1' = f_1' - f_{01}')$$

Using the estimates $\delta f' \leqslant O\,(f_m)$, $\delta f_1' \leqslant O\,(f_m)$, and (1) and remembering that when $\xi \to \infty$ only collisions of type $(\Gamma, X) \to (\Gamma, \Gamma)$ remain, we reduce the collision integral (2) to the form (d is the diameter of the molecule)

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